

Static Optimization with MATLAB

Day 3: Constrained Optimization with Mixed Constraints

GSEFM Pre-Semester Course

Luca Schmitz

Goethe University Frankfurt

September 22-28, 2026 | Frankfurt

Problem Setting I

Definition (Optimization Problem with Mixed Constraints)

An optimization problem with mixed constraints consists of a set of feasible solutions M , given as:

$$M = \{x \in \mathbb{R}^n \mid g(x) \leq 0, h(x) = 0\}.$$

where $g : \mathbb{R}^n \rightarrow \mathbb{R}^l$, $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and an objective function $f : M \rightarrow \mathbb{R}$. The goal is to find the maximum value of $f(x)$ over all $x \in M$, which is expressed as:

$$\max_{x \in M} f(x)$$

Problem Setting II

- We assume that f , the inequality constraints $g_1, \dots, g_I : \mathbb{R}^n \rightarrow \mathbb{R}$, and the equality constraints $h_1, \dots, h_I : \mathbb{R}^n \rightarrow \mathbb{R}$ are twice continuously differentiable.
- Basic Idea to tackle the problem: Combine everything we know so far.

The Karush-Kuhn-Tucker Problem I

We can also set up a Lagrangian for this problem:

$$\mathcal{L}(x, \lambda) = f(x) - \lambda' g(x) - \mu' h(x)$$

where λ and μ are vectors of multipliers. [» Different Conventions](#)

The Karush-Kuhn-Tucker Problem II

Definition (Karush-Kuhn-Tucker Point)

The following conditions are called Karush-Kuhn-Tucker conditions:

$$D_x \mathcal{L}(x, \lambda) = Df(x) - \lambda' Dg(x) - \mu' Dh(x) = \mathbf{0}$$

$$\text{for each } j = 1, \dots, l \quad g_j(x) \leq 0$$

$$\lambda_j \geq 0 \text{ non-negativity of LM}$$

$$\lambda_j g_j(x) = 0 \text{ complementary slackness condition}$$

$$h(x) = \mathbf{0}$$

A point $(x^*, \lambda^*, \mu^*) \in \mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R}^m$ satisfying above conditions is called a Karush-Kuhn-Tucker point (KKT-point).

The Karush-Kuhn-Tucker Theorem I

Theorem (The Karush-Kuhn-Tucker Theorem)

Let x^ be an optimal solution to the Karush-Kuhn-Tucker problem*

$$\max_{x \in \mathbb{R}^n} f(x) \quad \text{s.t. } g(x) \leq \mathbf{0}, h(x) = \mathbf{0}$$

where f , g , and h are continuously differentiable and $\text{rank}[Dg^b(x^), Dh(x^*)]' = B + m < n$ (LICQ/NDCQ). Then there exist uniquely determined Lagrange multipliers $\lambda^* \in \mathbb{R}^l$ and $\mu^* \in \mathbb{R}^m$ such that (x^*, λ^*, μ^*) is a KKT-point.*

The Karush-Kuhn-Tucker Theorem II

The LICQ/NDCQ is not the only condition that guarantees the existence of a KKT-point. There are other conditions that are sufficient for the existence of a KKT-point, e.g.

Constraint	Acronym	Statement
Linearity constraint qualification	LCQ	If g_i and h_j are affine functions , then no other condition is needed.
Linear independence constraint qualification	LICQ	The gradients of the active inequality constraints and the gradients of the equality constraints are linearly independent at x^* .
Mangasarian-Fromovitz constraint qualification	MFCQ	The gradients of the equality constraints are linearly independent at x^* and there exists a vector $d \in \mathbb{R}^n$ such that $\nabla g_i(x^*)^\top d < 0$ for all active inequality constraints and $\nabla h_j(x^*)^\top d = 0$ for all equality constraints. ^[10]
Constant rank constraint qualification	CRCQ	For each subset of the gradients of the active inequality constraints and the gradients of the equality constraints the rank at a vicinity of x^* is constant.
Constant positive linear dependence constraint qualification	CPLD	For each subset of gradients of active inequality constraints and gradients of equality constraints, if the subset of vectors is linearly dependent at x^* with non-negative scalars associated with the inequality constraints, then it remains linearly dependent in a neighborhood of x^* .
Quasi-normality constraint qualification	QNCQ	If the gradients of the active inequality constraints and the gradients of the equality constraints are linearly dependent at x^* with associated multipliers λ_j for equalities and $\mu_i \geq 0$ for inequalities, then there is no sequence $x_k \rightarrow x^*$ such that $\lambda_j \neq 0 \Rightarrow \lambda_j h_j(x_k) > 0$ and $\mu_i \neq 0 \Rightarrow \mu_i g_i(x_k) > 0$.
Slater's condition	SC	For a convex problem (i.e., assuming minimization, f, g_i are convex and h_j is affine), there exists a point x such that $h(x) = 0$ and $g_i(x) < 0$.

The Karush-Kuhn-Tucker Theorem III

It holds that

$$LICQ \Rightarrow MFCQ \Rightarrow CPLD \Rightarrow QNCQ$$

and

$$LICQ \Rightarrow CRCQ \Rightarrow CPLD \Rightarrow QNCQ$$

In practice weaker constraint qualifications are preferred since they apply to a broader selection of problems.

Sufficient Conditions for a Strict Local Maximum I

Theorem (Sufficient condition for a strict local maximum)

Let (x^*, λ^*, μ^*) a KKT point. Assume that the Hessian of the Lagrangian function with respect to the choice variables x , evaluated at (x^*, λ^*, μ^*) , given by

$$D_x^2 \mathcal{L}(x^*, \lambda^*) = D^2 f(x^*) - \lambda^{*'} D^2 g(x^*) - \mu^{*'} D^2 h(x^*),$$

is negative definite subject to the constraints $Dg^{b,>}(x^*)y = \mathbf{0}$ and $Dh(x^*)y = \mathbf{0}$, i.e., it holds that

$$y' D_x^2 \mathcal{L}(x^*, \lambda^*) y < 0, \text{ for all } y \in \mathbb{R}^n, y \neq 0 \text{ with } Dg^{b,>}(x^*)y = \mathbf{0} \\ \text{and } Dh(x^*)y = \mathbf{0}.$$

Then x^* is a strict local maximizer of f subject to $g(x) \leq \mathbf{0}$, $h(x) = \mathbf{0}$.

Sufficient Conditions for a Strict Local Maximum II

⇒ To check this condition we can use, as before, the bordered Hessian with the binding constraints with positive Lagrange Multiplier $g^{b,>}$ and the equality constraints h . Now, we have to look at the $n - m - B^>$ leading principal minors, where $B^> \leq B$ is the dimension of $g^{b,>}$.

Extension to Generalized Concave Problems I

Usually in order to conclude that a local maximum is also a global maximum, or we to omit checking the second order condition, we want to argue that we are dealing with a convex optimization problem, i.e., we need to show that

1. the objective function f is concave
2. the constraint set M , defined as

$$M = \{x \in \mathbb{R}^n \mid g(x) \leq 0, h(x) = 0\},$$

is convex.

Extension to Generalized Concave Problems II

How to check that M is convex?

- Standard setting: $g_j, j = 1, \dots, l$, are convex, $h_i, i = 1, \dots, m$, affine linear.
- We saw that we can reduce f to be pseudoconcave and $g_j, j = 1, \dots, l$, to be quasiconvex, and that $h_i, i = 1, \dots, m$, must be quasilinear (a function that is quasiconvex and quasiconcave).
- Recall that: concave $\Rightarrow$ pseudoconcave $\Rightarrow$ strictly quasiconcave $\Rightarrow$ quasiconcave (for a differentiable function)

For general nonlinear h , the set M is not convex, and therefore we do not have a convex optimization problem.

Exercise Mixed Constraints

$$\begin{aligned} & \max_{(x,y) \in \mathbb{R}^2} (x - y^2) \\ & \text{s.t. } x^2 + y^2 = 4 \\ & \quad x \geq 0, \ y \geq 0 \end{aligned}$$

which we can rewrite into:

$$\begin{aligned} & \max_{(x,y) \in \mathbb{R}^2} (x - y^2) \\ & \text{s.t. } \begin{aligned} h_1(x, y) &= x^2 + y^2 - 4 = 0 \\ g_1(x, y) &= -x \leq 0 \\ g_2(x, y) &= -y \leq 0 \end{aligned} \end{aligned}$$

Exercise Mixed Constraints Solution I

Set up the Lagrange function with μ being the multiplier of the equality constraint h and $\lambda_1, \lambda_2 \geq 0$ denoting the Lagrange multipliers of the inequality constraints g_1 and g_2 , respectively.

$$\mathcal{L}(x, y, \mu, \lambda_1, \lambda_2) = x - y^2 - \mu(x^2 + y^2 - 4) - \lambda_1(-x) - \lambda_2(-y)$$

According to the Kuhn-Tucker Theorem, the necessary conditions of this problem are:

1. FOC w.r.t x : $\frac{\partial \mathcal{L}}{\partial x}(x^*, y^*, \mu^*, \lambda_1^*, \lambda_2^*) = 1 - 2x^*\mu^* + \lambda_1^* = 0$
2. FOC w.r.t y : $\frac{\partial \mathcal{L}}{\partial y}(x^*, y^*, \mu^*, \lambda_1^*, \lambda_2^*) = -2y^* - 2y^*\mu^* + \lambda_2^* = 0$
3. equality constraint: $x^{*2} + y^{*2} - 4 = 0$
4. inequality constraints: $-x^* \leq 0, -y^* \leq 0$
5. positive multipliers: $\lambda_1^* \geq 0, \lambda_2^* \geq 0$
6. complementary slackness: $\lambda_1^*(-x^*) = 0, \lambda_2^*(-y^*) = 0$

Exercise Mixed Constraints Solution II

- From FOC 1 we can rearrange:

$$1 + \lambda_1^* = 2x^*\mu^*$$

Since $\lambda_1^* \geq 0$, the LHS, $1 + \lambda_1^*$, is strictly positive

⇒ RHS also has to be strictly positive: $2x^*\mu^* > 0$

⇒ $x^* \neq 0, \mu^* \neq 0$ and $2x^*\mu^* > 0$ can only hold for either $x^*, \mu^* > 0$ or $x^*, \mu^* < 0$.

By the inequality constraint $x^* \geq 0$, we conclude that $x^* > 0$. As a byproduct, we also get $\mu^* > 0$.

⇒ this requires $\lambda_1^* = 0$ from the complementary slackness condition.

Exercise Mixed Constraints Solution III

- Take 2nd FOC and rearrange

$$\lambda_2^* = 2y^*(1 + \mu^*)$$

If $\lambda_2^* > 0$, then complementary slackness requires that $y^* = 0$. Then, however, the above equation cannot hold.

- $\Rightarrow$ we must be in the case where $\lambda_2^* = 0$. Thus it has to hold that $0 = 2y^*(1 + \mu^*)$. This can be achieved either by $y^* = 0$ or $\mu^* = -1$. But above, we showed that $\mu^* > 0$.
- $\Rightarrow y^* = 0$.

Exercise Mixed Constraints Solution IV

- To sum up, we already got $x^* > 0$, $\lambda_1^* = 0$, $y^* = 0$, $\lambda_2^* = 0$
- Now we can almost solve as in the Lagrange case:
- From the binding constraint we get

$$x^{*2} = 4 \quad \Rightarrow \quad x^* = \pm 2$$

However, negative x are ruled out by the first inequality constraint, so we have $x^* = 2$

- Can use the 2nd FOC to solve for $\mu^* = \frac{1}{4}$.
 $\Rightarrow$ i.e. our solution candidate is

$$(x^*, y^*, \mu^*, \lambda_1^*, \lambda_2^*) = (2, 0, \frac{1}{4}, 0, 0)$$

Exercise Mixed Constraints Solution V

Let us check the Constraint Qualification (CQ):

With $x^* > 0$, the first inequality constraint is non-binding, therefore we only have to check the Jacobian of the equality h and the binding inequality constraint g_2 :

$$\begin{bmatrix} \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x^* & 2y^* \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

$\Rightarrow$ we have neither a zero-row, nor linearly dependent rows, so the rank of this Jacobian is equal to 2 and thus full and the CQ therefore fulfilled.

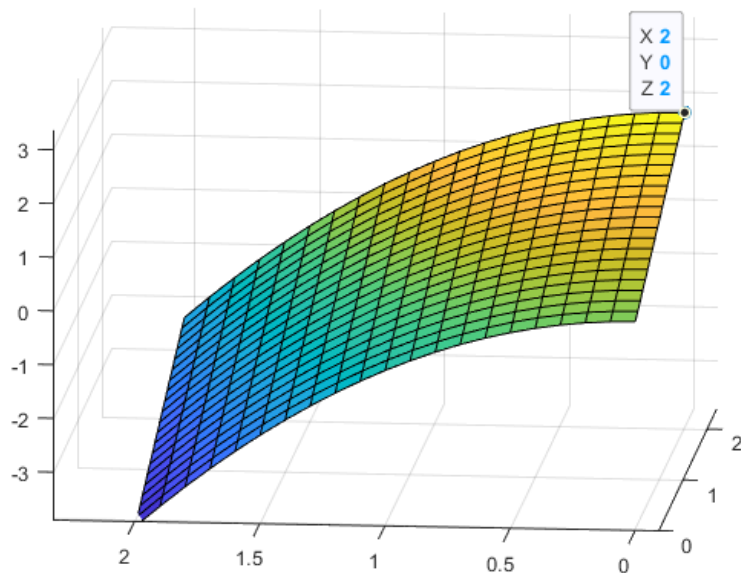
Exercise Mixed Constraints Solution VI

We check the Bordered Hessian. Note that the binding inequality constraint g_2 comes in pair with a **zero** Lagrange multiplier $\lambda_2^* = 0$, thus, we do not consider this binding constraint in the bordered Hessian, i.e., $B^> = 0$. Also, we only have one equality constraint, $m = 1$. Thus, we have to check the last leading principal minor, $n - m - B^> = 2 - 1 - 0 = 1$. The sign condition $(-1)^{m+B^>+1} = (-1)^2 = 1$ has to hold. The bordered Hessian is given by

$$H = \begin{bmatrix} 0 & 2x^* & 2y^* \\ 2x^* & -2\mu^* & 0 \\ 2y^* & 0 & -2 - 2\mu^* \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 4 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} \end{bmatrix}$$

The determinant of this matrix is $\det(H) = 40 > 0$ and thus we have a strict local maximum.

Exercise Mixed Constraints Solution VII



Economic Application I

- Consider a firm that produces two goods of quantity x_1 and x_2 . Inverse demand for the first is given by $p_1 = 100 - 1.5x_1$ and for the second one by $p_2 = 50 - 2x_2$.
- Production cost for both are equal and constant 10€ per unit.
- Because production causes emissions, the firm is subject to legal restrictions that only admit a maximum of total 33 units of production (sum of x_1 and x_2). Furthermore, capacity constraints restrict the production of each of the goods to be less than 28 units.

Economic Application II

1. Determine the optimal production decision (x_1, x_2) that maximizes the firm's total profit.
2. Give an interpretation of the result. What can be derived from the magnitude of the Lagrange multipliers?
3. Assume that the emission constraint is substituted by an emission tax of 12€ per unit produced. What is now the optimal production decision for (x_1, x_2) ?

Economic Application Solution I

1. Optimal Production decision:

First, the problem can be stated as

$$\begin{aligned}\Pi(x_1, x_2) &= p_1 x_1 + p_2 x_2 - 10(x_1 + x_2) \\ \text{s.t. } p_1 &= 100 - \frac{3}{2}x_1, \quad p_2 = 50 - 2x_2 \\ x_1 + x_2 &\leq 33 \\ x_1 &\leq 27, \quad x_2 \leq 27\end{aligned}$$

Plugging in the binding price equations into the objective function:

$$\Pi(x_1, x_2) = \left(100 - \frac{3}{2}x_1\right)x_1 + (50 - 2x_2)x_2 - 10(x_1 + x_2)$$

Economic Application Solution II

The Lagrangian is

$$\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2, \lambda_3) = \left(100 - \frac{3}{2}x_1\right)x_1 + (50 - 2x_2)x_2 - 10(x_1 + x_2) - \lambda_1[x_1 + x_2 - 33] - \lambda_2[x_1 - 27] - \lambda_3[x_2 - 27]$$

Kuhn-Tucker Conditions:

$$\frac{\partial \mathcal{L}}{\partial x_1}(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*, \lambda_3^*) = 100 - 3x_1^* - 10 - \lambda_1^* - \lambda_2^* = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2}(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*, \lambda_3^*) = 50 - 4x_2^* - 10 - \lambda_1^* - \lambda_3^* = 0$$

$$\lambda_1^* \geq 0, \quad \lambda_2^* \geq 0, \quad \lambda_3^* \geq 0$$

$$x_1^* + x_2^* - 33 \leq 0, \quad x_1^* \leq 27, \quad x_2^* \leq 27$$

$$\lambda_1(x_1^* + x_2^* - 33) = 0, \quad \lambda_2(x_1^* - 27) = 0, \quad \lambda_3(x_2^* - 27) = 0$$

Economic Application Solution III

- Because we have 3 constraints than can all be possibly binding or slack, we have $2^3 = 8$ different possibilities

g^1	g^2	g^3	x_1^*	x_2^*	λ^*	possible?
$x_1 + x_2 < 33$	$x_1 < 27$	$x_2 < 27$	30	10	(0,0,0)	violates g^1 and g^2
$x_1 + x_2 < 33$	$x_1 < 27$	$\lambda_3 > 0$	30	27	(0,0,-72)	violates g^1 and g^2
$x_1 + x_2 < 33$	$\lambda_2 > 0$	$x_2 < 27$	27	10	(0,9,0)	violates g^1
$x_1 + x_2 < 33$	$\lambda_2 > 0$	$\lambda_3 > 0$	27	27	(0,9,-68)	violates g^1
$\lambda_1 > 0$	$x_1 < 27$	$x_2 < 27$	26	7	(12,0,0)	yes
$\lambda_1 > 0$	$x_1 < 27$	$\lambda_3 > 0$	6	27	(72,0,-143)	negative multiplier
$\lambda_1 > 0$	$\lambda_2 > 0$	$x_2 < 27$	27	6	(16,-7,0)	negative multiplier
$\lambda_1 > 0$	$\lambda_2 > 0$	$\lambda_3 > 0$				too many binding constraints

$\Rightarrow$ Only one possible point $(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*, \lambda_3^*) = (26, 7, 12, 0, 0)$

Economic Application Solution IV

Check Constraint Qualification at this point: Only constraint g^1 is binding, thus we only have to look at its Jacobian.

$$Dg^1(x_1, x_2) = \begin{bmatrix} \frac{\partial g^1}{\partial x_1} & \frac{\partial g^1}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

which is clearly non-zero and the CQ therefore satisfied.

For sufficiency check the Bordered Hessian, again bordered only by the binding constraint:

$$\bar{H} = \begin{bmatrix} 0 & g_1^1 & g_2^1 \\ g_1^1 & \mathcal{L}_{11} & \mathcal{L}_{12} \\ g_2^1 & \mathcal{L}_{21} & \mathcal{L}_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -3 & 0 \\ 1 & 0 & -4 \end{bmatrix}$$

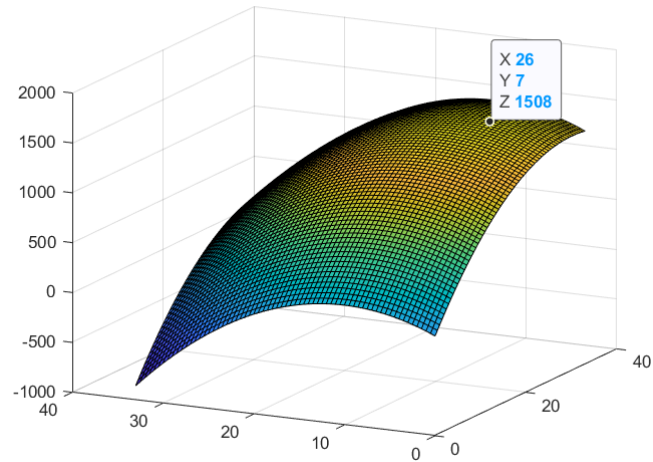
For a strict local maximum, the determinant of $\bar{H}$ has to have the sign of $(-1)^{m+1} = (-1)^2 = 1$, i.e. positive. It gives $|\bar{H}| = -(-4) + -(-3) = 7 > 0$

Economic Application Solution V

Sufficient Conditions and Uniqueness of the maximum:

- constraints are linear (affine) and thus quasiconcave
- objective function $\Pi(x_1, x_2) = 90x_1 - \frac{3}{2}x_1^2 + 40x_2 - 2x_2^2$ is strictly quasiconcave

Economic Application Solution VI



Economic Application Solution VII

2. Interpretation:

In essence, only constraint 1 is binding: $x_1^* + x_2^* = 33$, the other two constraints are slack, therefore, their shadow prices are zero. The shadow price of constraint 1 is $\lambda_1 = 12$, meaning that the firm would pay 12 monetary units to be able to produce one unit more of its products.

Economic Application Solution VIII

3. Tax instead of cap

The tax can be introduced in the profit function as an additional per-unit cost and the overall resource constraint is eliminated. This yields the new Lagrangian:

$$\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2) = \left(100 - \frac{3}{2}x_1\right)x_1 + (50 - 2x_2)x_2 - 22(x_1 + x_2) - \lambda_1[x_1 - 27] - \lambda_2[x_2 - 27]$$

Economic Application Solution IX

Now, the Kuhn-Tucker conditions yield:

$$\frac{\partial \mathcal{L}}{\partial x_1}(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*) = 100 - 3x_1^* - 22 - \lambda_1^* = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2}(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*) = 50 - 4x_2^* - 22 - \lambda_2^* = 0$$

$$\lambda_1^* \geq 0, \quad \lambda_2^* \geq 0$$

$$x_1^* \leq 27, \quad x_2^* \leq 27$$

$$\lambda_1^*(x_1^* - 27) = 0, \quad \lambda_2^*(x_2^* - 27) = 0$$

which gives 4 potential constraint combinations:

Economic Application Solution X

g^1	g^2	x_1^*	x_2^*	λ^*	possible?
$x_1 < 27$	$x_2 < 27$	26	7	(0,0)	yes
$x_1 < 27$	$\lambda_2 > 0$	26	27	(0,-80)	negative multiplier
$\lambda_1 > 0$	$x_2 < 27$	27	7	(-3,0)	negative multiplier
$\lambda_1 > 0$	$\lambda_2 > 0$	27	27	(-3,-80)	negative multipliers

Economic Application Solution XI

In optimum, no constraint is binding, so don't need to check the Non-degenerate Constraint Qualification $\Rightarrow$ basically back to unconstrained case.

Concerning sufficiency of a strict local maximum, check whether principal minors of the Hessian are of alternating sign:

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial x_1 \partial x_1} & \frac{\partial^2 \Pi}{\partial x_1 \partial x_2} \\ \frac{\partial^2 \Pi}{\partial x_2 \partial x_1} & \frac{\partial^2 \Pi}{\partial x_2 \partial x_2} \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}$$

$$|H_1| = -3 < 0, \quad |H_2| = 12 > 0 \Rightarrow \text{fulfilled}$$

Economic Application Solution XII

The unique maximum is $(x_1^*, x_2^*) = (26, 7)$ where none of the inequality constraints are binding.

As we can see, this is the same solution as above! The reason is that the shadow price from before, the Lagrange multiplier of 12 of the total quantity constraint, is replaced by a unit cost increase of also exactly 12. The firm would pay exactly 12 more units of profit to violate the constraint and produce more, and this is now exactly the increase in marginal unit cost. If we know the shadow price of the firm, we can achieve the same result with a marginal tax as with a quantity restriction.

References I

- Alt, W. (2011). *Nichtlineare optimierung eine einführung in theorie, verfahren und anwendungen: Eine einführung in theorie, verfahren und anwendungen*. Vieweg+Teubner Verlag.
- Chiang, A. C., & Wainwright, K. (2005). *Fundamental methods of mathematical economics*. McGraw-Hill Education.
- de la Fuente, A. (2000). *Mathematical methods and models for economists*. Cambridge University Press.
- Geiger, C., & Kanzow, C. (1999). *Numerische verfahren zur lösung unrestringierter optimierungsaufgaben (springer-lehrbuch)*. Springer.
- Geiger, C., & Kanzow, C. (2002). *Theorie und numerik restringierter optimierungsaufgaben*. Springer.
- Ulbrich, M. (2011). *Nichtlineare optimierung*. Birkhäuser Verlag.
- Werner, J. (2002). Unrestringierte optimierungsaufgaben [Last visited on 2023/09/19]. <http://num.math.uni-goettingen.de/werner/uncopt.pdf>
- Werner, J. (2012). Vorlesung über optimierung [Last visited on 2023/09/19]. <https://num.math.uni-goettingen.de/werner/optim.pdf>

Different Conventions for the Lagrangian

max $\mathcal{L}(x, \lambda) = f(x) - \lambda'g(x) - \mu'h(x)$ with $\lambda \geq 0, g(x) \leq 0, \dots$

min $\mathcal{L}(x, \lambda) = f(x) + \lambda'g(x) + \mu'h(x)$ with $\lambda \geq 0, g(x) \leq 0, \dots$

max $\mathcal{L}(x, \lambda) = f(x) + \lambda'g(x) + \mu'h(x)$ with $\lambda \geq 0, g(x) \geq 0, \dots$

min $\mathcal{L}(x, \lambda) = f(x) - \lambda'g(x) - \mu'h(x)$ with $\lambda \geq 0, g(x) \geq 0, \dots$

[▶ Go Back](#)